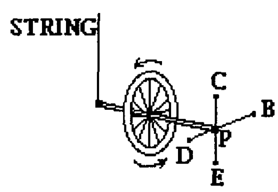
Physics 3
Fall 2009
Assignment 10
Due: in class Wed. 18 Nov

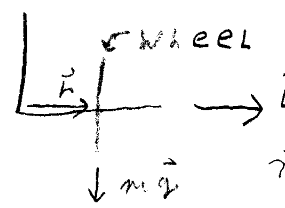
Read: Finish Ch.11 & start Ch. 13
Probs: Ch.11: 35, 37, 38, 42, 47, 51, 52

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

1) Which of the following is an accurate statement? 1) D

A) A particle moving in a straight line with constant speed necessarily has zero angular momentum.
B) Consider a planet moving in a circular orbit about a star. Even if the planet is spinning it is not possible for its total angular momentum to be zero.
C) If the speed of a particle is constant, then the angular momentum of the particle about any specific origin must also be constant.
D) The angular momentum of a moving particle depends on the specific origin with respect to which the angular momentum is calculated.
E) If the torque acting on a particle is zero about an arbitrary origin, then the angular momentum of the particle is also zero about that origin.





$\vec{\tau} = mgr \otimes \vec{in}$

2) In the figure, a rapidly spinning bicycle wheel is suspended from a string attached to the axle, as shown here. Initially the axis is held fixed in the position shown. When the axle is released, point P on the axle will: 2) ∴ B

A) remain where it is.
B) move toward point B.
C) move toward point C.
D) move toward point D.
E) move toward point E.

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11/14/09

Physics 3 : ps #10 Solutions

Ch 11 #s 35, 37, 38, 42, 47, 51, 52

#35 Weightlifter has 2 25kg masses on ends of 15 kg rod, 1.6 m long. He spins @ 10rpm.
Magnitude of barbell angular momentum:

$\vec{L} = \vec{r} \times \vec{p} = I\omega$, when motion is perpendicular

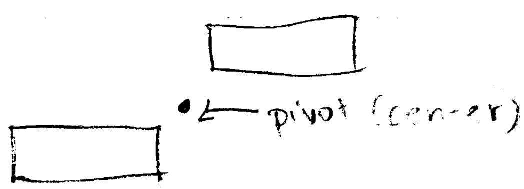
rod $I\omega = \left(\frac{1}{12}\right)(15\,kg)(10\,rpm) \rightarrow$ convert rpm to rad/sec
$= (1/12)(15\,kg)(1.05\,rad/sec)(1.6\,m)^2 = \boxed{2.1\ kg\ m^2/sec}$

weights $= \vec{r} \times \vec{p}\,(2) = I\omega(2) = (2)(25\,kg)(.8\,m)^2(1.05\,rad/sec) = \boxed{33.6\ kg\,m^2/sec}$

$\boxed{\text{Total angular momentum} = 35.7\ kg\ m^2/sec}$

#37 Two 1800kg cars traveling opposite each other @ 90 km/hr. CM = 3.0m from highway center.

Magnitude and direction of angular momentum of two-car system about point on highway centerline:



Equation: $\vec{L} = \vec{r} \times \vec{p} = \sum_i \vec{r}_i \times \vec{p}_i$, where P = angular momentum

$= \sum r_i \cdot p_i$ if vertical distance from car to road center $= r_i$

Need units in m and m/s → x2 for two car

Therefore: $L = (2)(3\text{ m})(1800\text{ kg})(25\text{ m/s})$

$\vec{L} = 2.7 \times 10^5$ kg·m²/sec out of the road
(can use right hand rule to see that both cars have same direction of angular momentum

$\boxed{\#38}$ 880-g (.880 kg) baseball bat w/ rotational inertia = .048 kg·m².
Bat swung so end moves 50 m/s

(a) angular momentum about pivot @ stationary end

$L = \vec{r} \times \vec{p} = I\omega$ → need I for end around pivot instead of cm (parallel axis theorem)

$I = I_{cm} + MR^2 = .048 + (.880)(.43\,m)^2$

$I = .21$

$I\omega = (.21\ kg \cdot m^2)(50\,m/s) / (.43 + .31)\,m = \boxed{14.2 = \vec{L}}$

For direction of angular momentum, use right hand rule:

$\boxed{\vec{L} = 14.2\ kg\,m^2/sec \text{ "out of the page"}}$

(b) constant torque applied to P to achieve this angular momentum in .25 sec.

Equation: $\vec{\tau} = \vec{r} \times \vec{F} = d\vec{L}/dt$

→ constant torque means that

$dL/dt = \Delta L / \Delta t = 35.8/.25$

$\boxed{\text{Torque} = 143.2\ kg\,m^2/s^2}$

#42 turntable initially motionless & frictionless with rotational inertia = .31 kg·m²
→ spinning 130 rpm
person holding wheel w/ rotational inertia = .22 kg·m²
she turns wheel upside down and turntable rotates 70 rpm

(a) person mass, where she is cylinder 30 cm in diameter

use conservation of angular momentum:

$I\omega_{\text{wheel initial}} = I\omega_{\text{wheel final}} + I\omega_{\text{turntable}} + I\omega_{\text{person}}$

$I_{(\text{wheel})}(\Delta\omega_{\text{wheel}}) = I\omega_{(\text{person})}$

$.22\ kg\cdot m^2\,(130\,rpm - (-130\,rpm)) = ((.31\ kg\cdot m^2) + (\tfrac{1}{2})MR^2_{\text{person}})(70\,rpm)$

→ change rpm to rad/sec

$.22\ kg\cdot m^2\,(27.2\ rad/sec) = [(.31\ kg\cdot m^2) + (\tfrac{1}{2})M(.15)^2](7.33\ rad/sec)$

solve for M:

$\boxed{M = 45\ kg}$

(b) Amt of work she does in turning wheel upside down

use conservation of energy to see what the "difference" is.
→ rotational energy isn't conserved b/c of work.

$\tfrac{1}{2}I\omega^2_{\text{initial}} = \tfrac{1}{2}I\omega^2_{\text{final}}$ (turntable, wheel and person)

$\tfrac{1}{2}(.22)(13.6)^2 = \tfrac{1}{2}(.22)(13.6)^2 + \tfrac{1}{2}(.31)(7.33)^2 + \tfrac{1}{2}(.5)(7.33)^2$

B/c of rotational energy $= \tfrac{1}{2}I\omega^2$ → direction of ω doesn't matter, and energy imbalance equals

$\tfrac{1}{2}(.31+.5)(7.33)^2 = \boxed{21.9\ kg\cdot m^2/sec^2}$

21.9 J work turned into rotational kinetic energy.

#47 turntable has rotational inertia $.021\ kg \cdot m^2$ rotating .29 rad/sec
wad of clay w/ 1.3 m/s horizontal velocity opposing rotation
→ radius = 15 cm from center
→ turntable slowed to .085 rad/sec

Mass of clay:

use conservation of angular momentum:

$$I\omega_i \text{ (turntable)} + \vec{r} \times \vec{p} \text{ (clay)} = I\omega \text{ (turntable + clay - final)}$$

$$(.021\ kg \cdot m^2)(.29\ rad/sec) + (.15\ m)(m)(-1.3\ m/s) = (.021\ kg \cdot m^2)(.085\ rad/sec)$$

solve for m: $\boxed{m = .022\ kg}$

Remember: use $\vec{r} \times \vec{p}$ for clay angular momentum because it is not initially in rotation, but it has relative angular momentum to turntable center.

Also, clay velocity is in opposite direction, so watch signs.

#51 upper stationary disk dropping onto lower rotational disk

~~upper~~ lower disk: 440g, r = 3.5cm → rotating 180 rpm

higher disk: initially stationary, 270g, r = 2.3 cm

If friction causes both disks to rotate together in the end,

(a) What is the end speed

→ conservation of momentum:

$I\omega$ top initial = $I\omega$ top and bottom final

$(.44\,kg)(\frac{1}{2})(.035\,m)^2(18.85\,rad/sec) = [(.44kg)(.035m)^2 + (.27kg)(.023m)^2](1/2)\,\omega_f$

(*) Solve for ω_f ⇒ $\boxed{\omega_f = 14.9\,rad/sec = \text{angular speed}}$

$= \boxed{142.3\ rev/min}$

(b) fraction of the initial kinetic energy lost to friction.

→ use conservation of energy to find the "difference"

$$\text{fraction conserved} = \frac{(\frac{1}{2})I\omega^2_{final}}{(\frac{1}{2})I\omega^2_{initial}}$$

(*) Remember $I = (\frac{1}{2})MR^2$ for disk

Fraction lost = 1 - fraction conserved

$$= 1 - \frac{(\frac{1}{2})(\frac{1}{2})(14.9)^2[(.44)(.035)^2 + (.27)(.023)^2]}{(\frac{1}{2})(\frac{1}{2})(18.85)^2(.44)(.035)^2}$$

$\boxed{\text{Fractional Energy lost} = .21\ (21\%)}$

$\boxed{\#52}$ spring on turntable with I = rotational inertia
turntable initially not moving
spring compressed distance Δx (m placed against it)
→ release causes block to move at right angle to turntable center and slide off turntable

Step #1

ⓐ linear speed of mass: conservation of energy

$$\left(\tfrac{1}{2}\right)kx^2 = \left(\tfrac{1}{2}\right)I\omega^2 + \left(\tfrac{1}{2}\right)mv^2$$
$$k\Delta x^2 = I\omega^2 + mv^2 \Longrightarrow \boxed{v^2 = \frac{k\Delta x^2 - I\omega^2}{m}}$$

Step #3

$$v^2 = \frac{k\Delta x^2 - I\left(\frac{mbv}{I}\right)^2}{m} \Longrightarrow mv^2 = k\Delta x^2 - (mbv)^2/I$$
$$v^2\left(m + \frac{m^2b^2}{I}\right) = k\Delta x^2$$
$$\boxed{v = \sqrt{\frac{k\Delta x^2}{m + \frac{m^2b^2}{I}}}}$$

ⓑ rotational speed of turntable: conservation of momentum
↳ angular momentum

Step #2

$$I\omega_{turntable} = m(R_{mass})^2\omega_{mass}$$

→ don't know ω_{mass}, but DO know that $R_{mass}\omega = v$ AND $R_{mass} = b$

$$I\omega_{turntable} = mbv$$
$$\boxed{\omega = \frac{mbv}{I}}$$

⟸ plug this back into above equation and solve for v w/o ω.